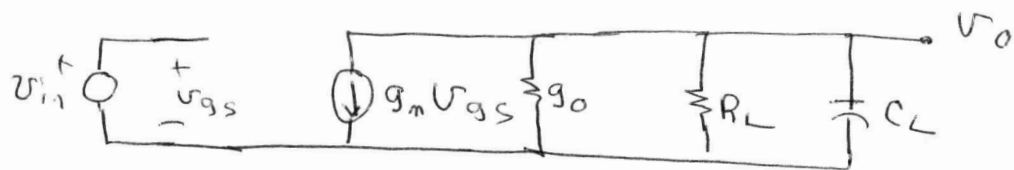


EE 330 Homework 13

Solutions Fall 2025

Problem 1

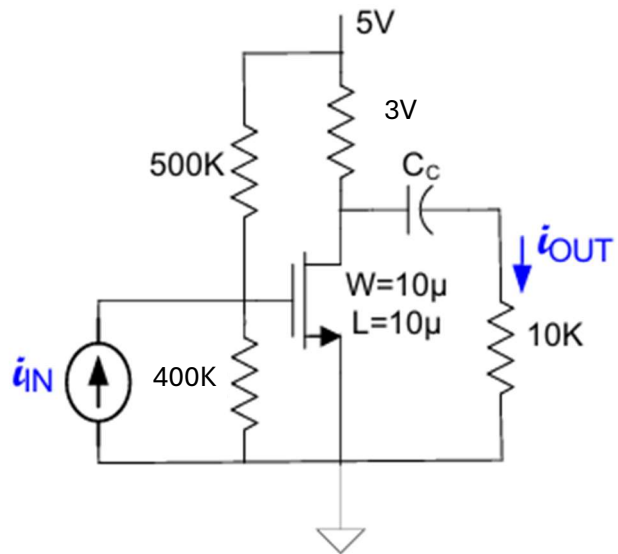


Summing currents on v_o node (with $G_L = 1/R_L$)

$$v_o (sC_L + G_L + g_o) + g_m v_i = 0$$

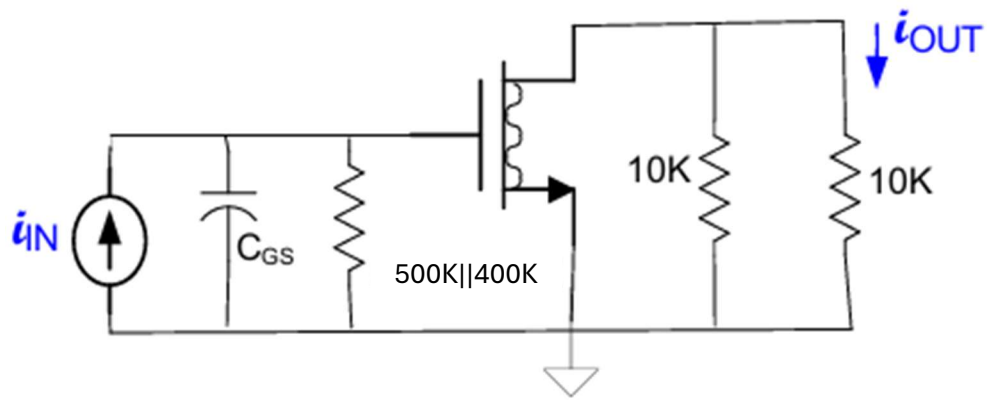
$$\therefore A_v(s) = \frac{v_o}{v_i} = \frac{-g_m}{sC_L + g_o + G_L} \approx \frac{-g_m}{sC_L + G_L}$$

Problem 2

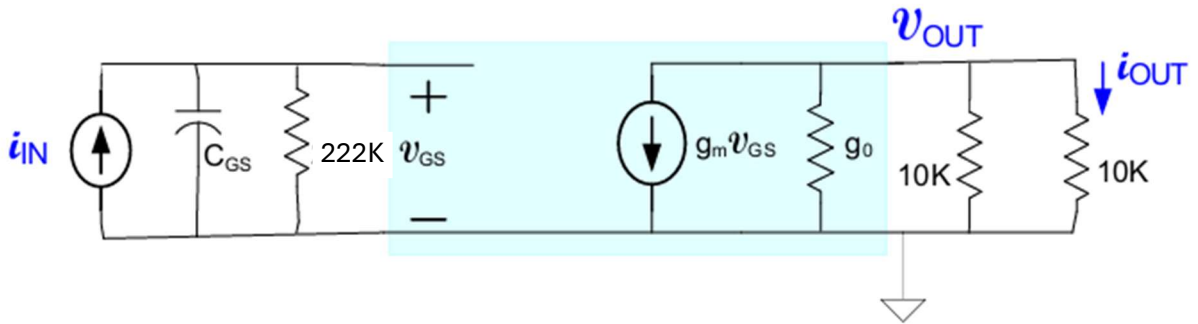


a)

Small signal equivalent circuit including C_{GS} capacitor



Including the MOS small signal model:



b)

Defining $R_B = 222K$, $G_B = 1/R_B$, and $G_L = 1/R_L = 1/10K$ and summing currents at input and output nodes obtain equations

$$V_{GS} (sC_{GS} + G_B) = I_{IN}$$

$$V_{OUT} (g_o + G_L + G_L) + g_m V_{GS} = 0$$

$$V_{OUT} G_L = I_{OUT}$$

Eliminating V_{OUT} and V_{GS} from these equations we obtain

$$\frac{I_{OUT}}{I_{IN}} = - \frac{g_m}{R_L (g_o + G_L + G_L)} \frac{1}{sC_{GS} + G_B} \approx - \frac{R_B g_m / 2}{sR_B C_{GS} + 1}$$

c)

Want to obtain

$$\left| \frac{R_B g_m / 2}{j\omega R_B C_{GS} + 1} \right| = 1$$

Which can be written as

$$\frac{(R_B g_m / 2)^2}{1 + (\omega R_B C_{GS})^2} = 1$$

Solving for angular frequency ω we obtain

$$\omega = \frac{\sqrt{(R_B g_m / 2)^2 - 1}}{R_B C_{GS}} \approx \frac{g_m}{2C_{GS}}$$

It remains to obtain g_m and C_{GS} . Observe by voltage divider $V_{GSQ}=1.33V$. So

$$g_m = \mu C_{ox} \frac{W}{L} (V_{GSQ} - V_{TH}) = 232.5 \mu S$$

$$C_{GS} = C_{ox} WL = 400 fF$$

So, unity gain frequency is approximately 290.625M rad/sec or 46.25 MHz

Problem 3 $I_{D2} = \left(\frac{W_2}{W_1} \frac{L_1}{L_2} \right) I_{D1} = (3)(50 \mu A) = 150 \mu A$

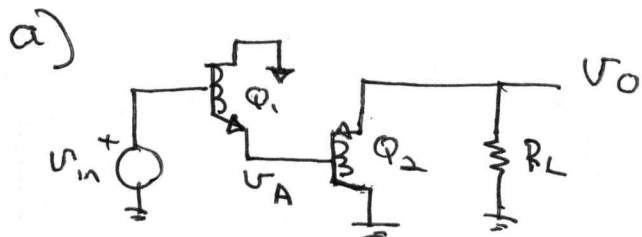
$$V_0 = 8V - I_{D2} \cdot R = 8V - (150 \mu A)(25k) = 4.25V$$

Problem 4 From Lecture Notes $A_v = -\frac{g_{m1}}{2g_{o1}} \cdot \beta = -\frac{V_{AF}}{2V_t} \beta$

$$\text{So } A_v \approx \frac{-200}{50 \text{ mV}} \cdot 100 = -400,000$$

Problem 5 Define $G_L = \frac{1}{R_L} = \frac{1}{1k\Omega}$

Observe this is a cascade of two CC stages



$$A_v = \frac{v_o}{v_{in}} \cdot \frac{v_A}{v_i}$$

$$\frac{v_o}{v_A} = \frac{+g_{m2}}{g_{m2} + G_L}$$

$$\frac{v_A}{v_{in}} = \frac{+g_{m1}}{g_{m1} + \frac{G_L}{\beta_2}}$$

$$\therefore A_v = \left(\frac{g_{m2}}{g_{m2} + G_L} \right) \left(\frac{g_{m1}}{g_{m1} + \frac{G_L}{\beta_2}} \right) \approx 1$$

b) $v_{inQ} = 0V$, $v_{oQ} = v_{inQ} - 0.6 + 0.6 = 0V$

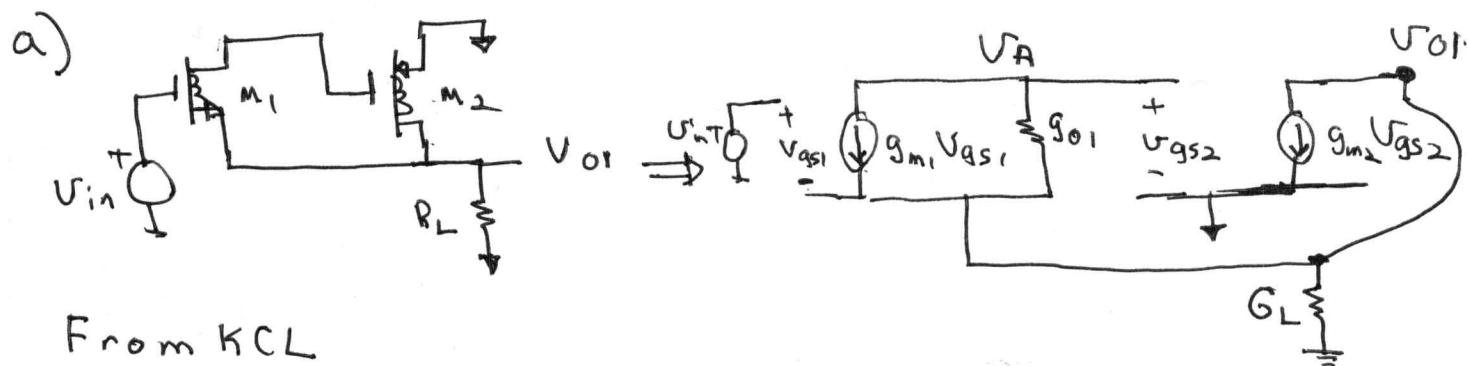
c) $R_{in} = r_{\pi 1} + \beta_1 R_{in2} = r_{\pi 1} + \beta_1 \beta_2 R_L \approx \beta_1 \beta_2 R_L$

d) If current sources ideal

$$v_{oMAX} = V_{CC}$$

$$v_{oMIN} = V_{EE} + 0.6V$$

Problem 6 Define $G_L = \frac{1}{R_L} = \frac{1}{1K\Omega}$

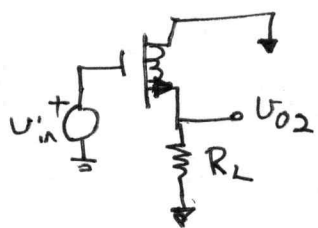


From KCL

$$\left. \begin{aligned} V_{o1}(g_{o1} + G_L) + g_{m2}V_A &= g_{m1}(V_{in} - V_{o1}) + g_{o1}V_A \\ V_A &= V_{o1} - \frac{g_{m1}}{g_{o1}}(V_{in} - V_{o1}) \end{aligned} \right\}$$

eliminating V_A , obtain

$$\frac{V_{o1}}{V_{in}} = \frac{g_{m1}g_{o1} + g_{m1}g_{m2}}{g_{o1}[g_{m2} + G_L] + g_{m1}g_{m2}} \approx \frac{g_{m1}g_{m2}}{g_{m1}g_{m2}}$$



Recognize as common drain amplifier

$$\therefore \frac{V_{o2}}{V_{in}} = \frac{g_{m1}}{g_{m1} + G_L}$$

b) $\frac{V_{o1}}{V_{in}} \approx 1$ to find $\frac{V_{o2}}{V_{in}}$, need g_{m1} . First obtain I_{DQ}

$$I_{DQ} = \mu C_{ox} \frac{W}{2L} (V_{inQ} - I_{DQ}R - V_{TH})^2$$

solving this equation for I_D , obtain

$$I_{DQ} = 58.7 \mu A \Rightarrow g_m = \sqrt{\mu C_{ox} \frac{W}{L} 2I_{DQ}} = 383 \frac{\mu A}{V}$$

thus

$$\frac{V_{o2}}{V_{in}} = \frac{383 \mu}{383 \mu + \frac{1}{5K}} = 0.657 \frac{V}{V}$$

c) Need to reduce $I_{B1} = 5 \mu A$, $I_{B2} = 10 \mu A$

For circuit on left

$$I_{B2} = \frac{\mu C_{ox} W}{2L} (V_{INQ} - V_{OQ} - V_{TH})^2$$

$$10 \mu A = 250 \mu \left(\frac{10}{4}\right) (1V - V_{OQ} - .4 V)^2$$

solving, obtain $V_{OQ} = 0.474 V$

For circuit on right, found in part b), I_{OQ}

$$\therefore V_{OQ} = (58.7 \mu)(5k) = 0.294 V$$

d) For $V_{INQ} = 4V$, circuit on left has

$$V_{OQ} = 3.47 V$$

For circuit on right, must again solve

$$I_{DQ} = \frac{\mu C_{ox} W}{2L} (V_{INQ} - I_{DQ} R - V_{TH})^2 \quad \text{for } I_{DQ}$$

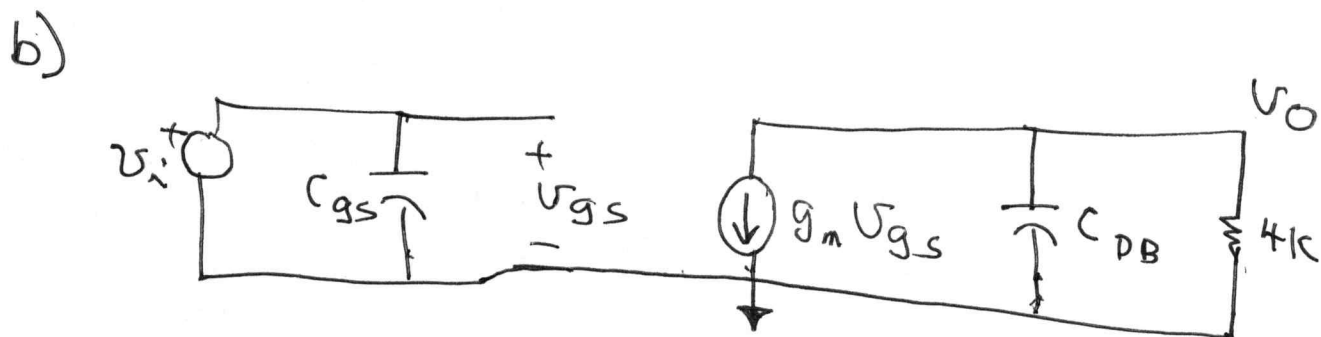
with $V_{INQ} = 4V$, obtain $I_{DQ} = 535 \mu A$

$$\text{so } V_{OQ} = (I_{DQ})(5k) = 2.67 V$$

Problem 7 From Lecture slides

$$A_v \approx -\frac{g_{m1}}{g_{o1}} = -\frac{\frac{I_{DQ}}{V_t}}{\frac{I_{DQ}}{V_{AF}}} = \frac{V_{AF}}{V_t} = -\frac{100}{25mV} = -4000$$

Problem 8



Summing currents at output node with $G_L = \frac{1}{4k}$

$$v_o (s C_{DB} + G_L) + g_m v_i = 0$$

$$\therefore \frac{v_o}{v_{in}} = \frac{-g_m}{s C_{DB} + G_L}$$

c)

$$\omega_{3dB} = \frac{G_L}{C_{DB}} \Rightarrow f_{3dB} = \frac{1}{(2\pi) R_L C_{DB}}$$

$$C_{DB} = C_{swD} \cdot 52\mu + C_{BOT} \cdot 120\mu^2$$

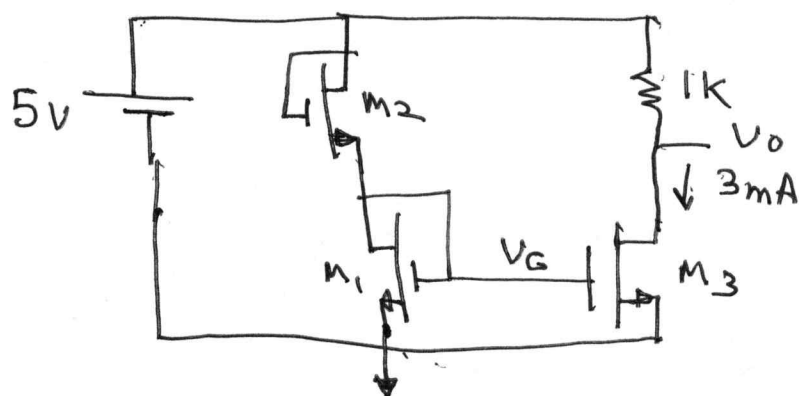
From table $C_{swD} = 212 \text{ af}/\mu$

$$C_{BOT} = 942 \text{ af}/\mu^2$$

$$\therefore C_{DB} = 11.0 \text{ fF} + 113 \text{ fF} = 124 \text{ fF}$$

So $f_{3dB} = \frac{1}{(2\pi)(4k)(124 \text{ fF})} = 320 \text{ MHz}$

Problem 9 One solution



Since $V_O = 2V$, want $V_G < 2V + V_{TH}$ to maintain saturation of M_3 .

So will set $V_G = 2V$

$$I_{D3} = \frac{\mu C_{ox} W}{2L} (2 - 0.4)^2$$

$$3E-3 = \frac{250\mu}{2} \cdot \frac{W_3}{L_3} (1.6)^2$$

$$\therefore \frac{W_3}{L_3} = 9.38 \quad \text{Let } L_3 = 1\mu, W_3 = 9.38\mu$$

Let $\frac{W_1}{L_1} = 9.38$ so unity mirror gain

$$\text{Let } L_1 = 1\mu, W_1 = 9.38\mu$$

$$\therefore I_{D1} = 3mA$$

Consider now M_2 . which also has $I_D = 3mA$

$$\therefore 3mA = \frac{\mu C_{ox} W_2}{2L_2} (5 - V_G - V_{TH})^2$$

$$3mA = \frac{250\mu}{2} \frac{W_2}{L_2} (5 - 2 - 0.4)^2$$

solving

$$\frac{W_2}{L_2} = 3.55 \quad \text{Let } L_2 = 10\mu, W_2 = 35.5\mu$$